Measurement Efficiency and n-Shot Readout of Spin Qubits

Hans-Andreas Engel,[1] Vitaly N. Golovach,[1] Daniel Loss,[1] L. M. K. Vandersypen,[2] J. M. Elzerman,[2] R. Hanson,[2] and L. P. Kouwenhoven[2]

[1]Department of Physics and Astronomy, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland

[2]Department of NanoScience and ERATO Mesoscopic Correlation Project, P.O. Box 5046, 2600 GA Delft, The Netherlands

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We consider electron spin qubits in quantum dots and define a measurement efficiency e to characterize reliable measurements via n-shot readouts. We propose various implementations based on a double dot and a quantum point contact (QPC) and show that the associated efficiencies e vary between 50% and 100%, allowing single-shot readout in the latter case. We model the readout microscopically and derive its time dynamics in terms of a generalized master equation, calculate the QPC current, and show that it allows spin readout under realistic conditions.

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The readout of a qubit state is of central importance for quantum information processing [1]. In special cases, the qubit state can be determined in a single measurement, referred to as single-shot readout. In general, however, the preparation and measurement need to be performed not only once but n times, where n depends on the qubit, the efficiency e of the measurement device, and on the tolerated inaccuracy (infidelity) α. In the first part of this Letter, we analyze such n-shot readouts for general qubit implementations and derive a lower bound on n in terms of e and α. We then turn to spin-based qubits and GaAs quantum dots [2,3] and analyze their n-shot readout based on a spin-charge conversion and charge measurement via quantum point contacts.

n-shot readout and measurement efficiency e.—How many times n do the preparation of a qubit in the *same* initial state and subsequent measurement need to be performed until the state of the qubit is known with some given infidelity α (n-shot readout)? We model the measurement process with a set of positive operator-valued measure (POVM) operators [1,4], $E_{A_0} = p_0|0\rangle \times \langle 0| + (1-p_1)|1\rangle\langle 1|$ and $E_{A_1} = (1-p_0)|0\rangle\langle 0| + p_1|1\rangle\langle 1|$, where p_0 and p_1 are probabilities. These operators describe measurements with outcomes A_0 and A_1, respectively. This POVM model can be pictured as follows. First, the qubit is coupled to some other device (e.g., to a reference dot, see below). Then this coupled system is measured and thereby projected onto some internal state. That state is accessed via an external "pointer" observable $\hat{A}$ [4] (e.g., a particular charge distribution, a time-averaged current, or noise). We assume that only two measurement outcomes are possible, either A_0 or A_1, which are classically distinguishable [5]. For initial qubit state $|0\rangle$ the expectation value is $\langle \hat{A}\rangle_0 = p_0 A_0 + (1-p_0)A_1$, while for initial state $|1\rangle$ it is $\langle \hat{A}\rangle_1 = (1-p_1)A_0 + p_1 A_1$. Let us take an initial qubit state $|0\rangle$ and consider a single measurement. With probability p_0, the measurement outcome is A_0 which one would interpret as "qubit was in state $|0\rangle$." However, with probability $1-p_0$, the outcome is A_1 and one might incorrectly conclude that "qubit was in state $|1\rangle$." Conversely, the initial state $|1\rangle$ leads with probability p_1 to A_1 and with $1-p_1$ to A_0. We now determine n for a given α, for a qubit either in state $|0\rangle$ *or* $|1\rangle$ (no superposition allowed [6]). For an accurate readout we need, roughly speaking, that $\langle \hat{A}\rangle_0$ and $\langle \hat{A}\rangle_1$ are separated by more than the sum of the corresponding standard errors. More precisely [7], we consider a parameter test of a binomial distribution of the measurement outcomes, one of which is A_0 with probability p. The null hypothesis is that the qubit is in state $|0\rangle$, thus $p = p_0$. The alternative is a qubit in state $|1\rangle$, thus $p = 1-p_1$. For sufficiently large n, namely $n\, p_{0,1}(1-p_{0,1}) > 9$, one can approximate the binomial with a normal distribution [8]. The qubit state can then be determined with significance level ("infidelity") α for

$$n > z_{1-\alpha}^2\left(\frac{1}{e} - 1\right), \tag{1}$$

$$e = \left[\sqrt{p_0 p_1} - \sqrt{(1-p_0)(1-p_1)}\right]^2, \tag{2}$$

with the quantile (critical value) $z_{1-\alpha}$ of the standard normal distribution function, $\Phi(z_{1-\alpha}) = 1 - \alpha = \frac{1}{2}[1 + \mathrm{erf}(z_{1-\alpha}/\sqrt{2})]$. We interpret e as *measurement efficiency*. Indeed, it is a single parameter $e \in [0,\ 1]$ which tells us if n-shot readout is possible. For $p_0 = p_1 = 1$, the efficiency is maximal, $e = 100\%$, and single-shot readout is possible ($n = 1$). Conversely, for $p_1 = 1 - p_0$ (e.g., $p_0 = p_1 = \frac{1}{2}$), the state of the qubit cannot be determined, not even for an arbitrarily large n, and the efficiency is $e = 0\%$. For the intermediate regime, $0\% < e < 100\%$, the state of the qubit is known after several measurements, with n satisfying Eq. (1) [9].

Visibility v.—When coherent oscillations between $|0\rangle$ and $|1\rangle$ are considered, the amplitude of the oscillating signal is $|\langle \hat{A}\rangle_1 - \langle \hat{A}\rangle_0|$, i.e., smaller than the value $|A_1 - A_0|$ by a factor of $v = |p_0 + p_1 - 1|$. Thus, we can take v as a measure of the visibility of the coherent oscillations.

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With v and the shift of the oscillations, $s = \frac{1}{2}(p_1 - p_0) = \frac{1}{2}(\langle\hat{A}\rangle_0 + \langle\hat{A}\rangle_1 - A_0 - A_1)/(A_1 - A_0)$, we can get e. Next, we see that $e = 0$, $v = 0$, and $p_1 = 1 - p_0$ are equivalent statements. We find the general relation $v^2 \leq e \leq v$, where the left inequality becomes exact for $p_0 = p_1$ and the right for $p_0 \in \{0, 1\}$ and/or $p_1 \in \{0, 1\}$; otherwise the inequalities are strict if $e, v > 0$. In particular, for p_0 and p_1 close to $\frac{1}{2}$ with $p_0 = p_1$, we see that the efficiency e can be much smaller than the visibility v.

Single spin readout.—We now discuss several concrete readout setups and their measurement efficiency. We consider a promising qubit, an electron spin confined in a quantum dot [2,3]. For reading out such a spin qubit, the time scale is limited by the spin-flip time T_1, which has a lower bound of $\approx 100\ \mu$s [10,11] (while T_2 is not of relevance here). One setup proposed in Ref. [2] is readout via a neighboring paramagnetic dot, where the qubit spin nucleates formation of a ferromagnetic domain. This leads to $p_0 = p_1 = \frac{3}{4}$ and thus $e = 25\%$. Another idea is to transfer the qubit information from spin to charge [2,3,12–14]. For this, we propose to couple the qubit dot to a second ("reference") dot [15] and discuss several possibilities how that coupling can be made spin dependent; see Fig. 1. The resulting charge distribution on the double dot will then depend on the qubit state and can be detected with an electrometer, such as a quantum point contact (QPC) [16–18] (see Fig. 1) or a single-electron transistor (SET) [19]. Single charges were detected on a time scale of 1 μs [19], which is much smaller than T_1.

Readout with different Zeeman splittings.—First, we propose a setup where efficiencies up to 100% can be reached; see Fig. 1(a). We take a double dot with different Zeeman splittings, $\Delta_z^{L,R} = E_{L,R}^{\downarrow} - E_{L,R}^{\uparrow}$, in each dot [20] and consider a single electron on the double dot. For initial qubit state $|\!\uparrow\rangle$, the electron can tunnel from state $|L_\uparrow\rangle \hat{=} \textcircled{$\uparrow$}_L \bigcirc_R$ to state $|R_\uparrow\rangle \hat{=} \bigcirc_L \textcircled{$\uparrow$}_R$ and vice versa, and analogously for qubit state $|\!\downarrow\rangle$. We consider time scales shorter than T_1, thus the states with different spins are not coupled. Next, we define the detunings $\varepsilon_{\uparrow,\downarrow} = E_L^{\uparrow,\downarrow} - E_R^{\uparrow,\downarrow}$, which are different for the up and down states, $\varepsilon_\downarrow - \varepsilon_\uparrow = \Delta_z^L - \Delta_z^R \neq 0$. The stationary state of the double dot depends on $\varepsilon_{\uparrow,\downarrow}$ and so does the QPC current $\bar{I}_{\uparrow,\downarrow}$ [we show this below, see Eq. (5) and $\bar{I}_{\rm incoh}$]. Therefore, initial states $|\!\uparrow\rangle$ and $|\!\downarrow\rangle$ can be identified through distinguishable stationary currents [5,21], $\bar{I}_\uparrow \neq \bar{I}_\downarrow$, *thus* $e = 100\%$ *and single-shot readout is possible.*

Spin-dependent tunneling provides another readout scheme, see Fig. 1(b), which we describe with spin-dependent tunneling amplitudes $t_d^{\uparrow,\downarrow}$. For $t_d^{\downarrow} \ll t_d^{\uparrow}$, only spin $\uparrow$ tunnels onto the reference dot while tunneling of spin $\downarrow$ is suppressed. We assume the same Zeeman splitting in both dots and resonance $\varepsilon = 0$. It turns out [Eq. (5)] that $\bar{I}_{\uparrow,\downarrow}$ depends on $t_d^{\uparrow,\downarrow}$ and thus the state of the qubit can be measured. However, the decay to the stationary state is quite slow in case the qubit is $|\!\downarrow\rangle$, due to the suppressed tunneling amplitude $t_d^{\downarrow}$. Since the difference in charge distribution between qubit $|\!\uparrow\rangle$ and $|\!\downarrow\rangle$ is larger at short time scales, it can thus be advantageous to measure the time-dependent current (discussed toward the end).

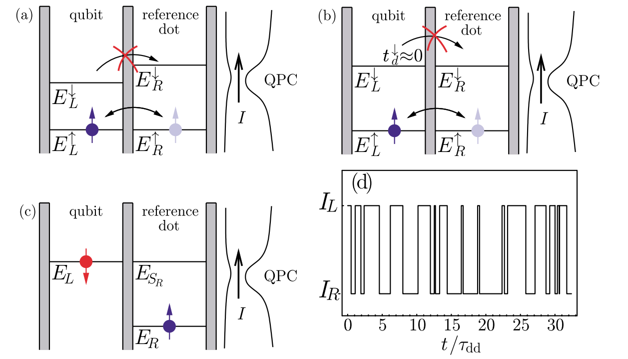

FIG. 1 (color online). Electron spin readout setup consisting of a double dot. The right "reference" dot is coupled capacitively to a QPC shown on the right. (a) Readout using different Zeeman splittings. For $\uparrow$, the electron tunnels between the two dots. For $\downarrow$, tunneling is suppressed by the detuning and the stationary state has a large contribution of the left dot since it has lower energy. This allows single-shot readout, i.e., $e = 100\%$. (b) Spin-dependent tunneling amplitudes, $t_d^{\downarrow} < t_d^{\uparrow}$, also enable efficient readout. (c) Readout with the singlet state. Tunneling of spin $\uparrow$ to the reference dot is blocked due to the Pauli principle. (d) Schematic current vs time during a single measurement. Here, τ_{dd} is the time scale for tunneling and we assume $\Gamma_{\rm tot} > t_d$, i.e., that the tunneling events can be resolved in the current.

Readout with Pauli principle.—We now consider the case where the reference dot contains initially an electron in spin up ground state; see Fig. 1(c). We assume gate voltages such that there are either two electrons on the right dot or one electron on each dot. Thus, we consider the five dimensional Hilbert space $|S_R\rangle \hat{=} \bigcirc_L \textcircled{$\uparrow\downarrow$}_R$, $|\!\uparrow\downarrow\rangle \hat{=} \textcircled{$\uparrow$}_L \textcircled{$\downarrow$}_R$, $|\!\downarrow\uparrow\rangle \hat{=} \textcircled{$\downarrow$}_L \textcircled{$\uparrow$}_R$, $|T_+\rangle \hat{=} \textcircled{$\uparrow$}_L \textcircled{$\uparrow$}_R$, $|T_-\rangle \hat{=} \textcircled{$\downarrow$}_L \textcircled{$\downarrow$}_R$, We define the "delocalized" singlet $|S_{LR}\rangle = (|\!\uparrow\downarrow\rangle - |\!\downarrow\uparrow\rangle)/\sqrt{2}$ and the triplet $|T_0\rangle = (|\!\uparrow\downarrow\rangle + |\!\downarrow\uparrow\rangle)/\sqrt{2}$. In the absence of tunneling, the corresponding energies are $E_{S_R} = 2\epsilon_R + U$ and $E_{S_{LR}} = E_{T_{0,\pm}} = \epsilon_L + \epsilon_R$ with charging energy U and single particle energies $\epsilon_{L,R}$. We can neglect states with two electrons on the qubit dot and the triplet states with two electrons on the reference dot, since they have a much larger energy (their admixture due to tunneling is small). We denote the state with an "extra" electron on the right dot as $|R\rangle \equiv |S_R\rangle$ with corresponding QPC current I_R. For state $|L\rangle \equiv |S_{LR}\rangle$ and for all triplet states, $|T_{0,\pm}\rangle$, the current is I_L. When tunneling is switched on and the qubit is initially in state $|\!\uparrow\rangle$, tunneling to the reference dot is blocked due to the Pauli exclusion principle [22]. Thus, the double dot will remain in the (stationary) state $|T_+\rangle\langle T_+|$ and the current in the QPC remains $\langle I\rangle = I_L$ (a so-called nondemolition measurement). On the other hand, for an initial qubit state $|\!\downarrow\rangle$, the initial state of the double dot is

$|\downarrow\uparrow\rangle = (|T_0\rangle - |S_{LR}\rangle)/\sqrt{2}$. The contribution $|S_{LR}\rangle$ of this superposition is tunnel coupled to $|S_R\rangle$ and will decay to the stationary state, which we describe by the density matrix $\bar{\rho}$, with corresponding QPC current $\bar{I}$ (see below for an explicit evaluation). In contrast, the triplet contribution $|T_0\rangle$ is not tunnel coupled to $|S_R\rangle$ due to spin conservation and does not decay. In total, the density matrix of the double dot decays into the stationary value $\frac{1}{2}(|T_0\rangle\langle T_0| + \bar{\rho})$. For $\varepsilon = 0$, the ensemble-averaged QPC current for qubit $|\downarrow\rangle$ is $\langle I\rangle = \frac{1}{2}(I_L + \bar{I}) \approx \frac{1}{4}(3I_L + I_R)$ and can thus be distinguished from I_L for qubit $|\uparrow\rangle$. However, in a single run of such a measurement, an initial qubit $|\downarrow\rangle$ decays either into $|T_0\rangle\langle T_0|$ or into $\bar{\rho}$, with 50% probability each. Since $|T_0\rangle\langle T_0|$ and $|T_+\rangle\langle T_+|$ lead to the same QPC current I_L, these two states are not distinguishable within this readout scheme and single-shot readout is not possible. The readout can now be described with the POVM model given above, with $|\uparrow\rangle \equiv |0\rangle$ and $|\downarrow\rangle \equiv |1\rangle$ and $A_\uparrow = I_L$; $A_\downarrow = \bar{I}$; $p_\uparrow = 1$; and $p_\downarrow = \frac{1}{2}$. Thus, the *measurement efficiency is* $e = 50\%$, i.e., to achieve a fidelity of $1 - \alpha = 99\%$, we need $n \geq 7$ readouts [8].

An analogous readout is possible if the ground state of the reference dot is a triplet, say $|RT_+\rangle \hat{=} \bigcirc_L \textcircled{\Uparrow}_R$. Using the same argument as above, we find again $e = 50\%$ [23].

Readout model.—So far we have introduced various spin readout schemes and the corresponding measurement efficiencies. In order to evaluate the signal strength $A_0 - A_1$ for these schemes, we now calculate the stationary charge distribution $\bar{\rho}$ and QPC current $\bar{I}$ for the case when the electron can tunnel coherently between the two dots (as a function of the detuning and the tunnel coupling). We describe the readout setup with the Hamiltonian $H = H_d + V_d + H_{\rm QPC} + V$. Here, $H_{\rm QPC}$ contains the energies of the (uncoupled) Fermi leads of the QPC. Further, H_d describes the double dot in the absence of tunneling, including orbital and electrostatic charging energies, $H_d|n\rangle = E_n|n\rangle$. It thus contains $\varepsilon = E_L - E_R$, the detuning of the tunneling resonance. The interdot tunneling Hamiltonian is defined as $V_d = t_d(|R\rangle \times \langle L| + |L\rangle\langle R|)$. (Note that for tunneling between $|S_{LR}\rangle$ and $|S_R\rangle$, t_d is $\sqrt{2}$ times the one-particle tunneling amplitude, since both states $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ are involved). V is a tunneling Hamiltonian describing transport through the QPC. The tunneling amplitudes, t_L^Q and t_R^Q, will be influenced by electrostatic effects, in particular, by the charge distribution on the double dot. Thus, we model the measurement of the dot state via the QPC with $V = (t_L^{\rm Q}|L\rangle \times \langle L| + t_R^{\rm Q}|R\rangle\langle R|)\sum(c_{\rm in}^\dagger c_{\rm out} + {\rm H.c.})$ [24–26]. Here, $c_{\rm in}^\dagger$ and $c_{\rm out}^\dagger$ create electrons in the incoming and the outgoing leads of the QPC, where the sum is taken over all momentum and spin states. We derive the master equation for the reduced density matrix ρ of the double dot, using standard techniques and making a Born-Markov approximation in V [27]. We allow for an arbitrary interdot tunnel coupling, i.e., we keep V_d exactly, with energy splitting $E = \sqrt{4t_d^2 + \varepsilon^2}$ in the eigenbasis of $H_d + V_d$. We obtain the master equation

$$\dot{\rho}_L = -\dot{\rho}_R = 2t_d{\rm Im}[\rho_{RL}], \tag{3}$$

$$\dot{\rho}_{RL} = \left[it_d + t_d\frac{\Gamma_Q\varepsilon}{E^2}(g_\Sigma - 2g_0)\right](\rho_R - \rho_L) - \frac{t_d\Gamma_Q}{\Delta\mu} - (\kappa\Gamma_Q + \Gamma_i - i\varepsilon)\rho_{RL}, \tag{4}$$

for $\rho_n = \langle n|\rho|n\rangle$ and $\rho_{RL} = \langle R|\rho|L\rangle$. In comparison to previous work [24–26], we find an additional term, $-t_d\Gamma_Q/\Delta\mu$, which comes from treating V_d exactly. We find that the current through the QPC is $I_L = 2\pi\nu^2 e\Delta\mu|t_L^Q|^2$ for state $|L\rangle$ and analogously I_R for state $|R\rangle$, and we choose $I_L, I_R \geq 0$. Here, $\Delta\mu > 0$ is the applied bias across the QPC and ν is the DOS at the Fermi energy of the leads connecting to the QPC. We define $g_\pm = g(\Delta\mu \pm E)$, $g_\Sigma = g_+ + g_-$, and $g_0 = g(\Delta\mu)$ with $g(x) = x/\Delta\mu(e^{x/kT} - 1)$. The values $g_{\pm,\Sigma,0}$ vanish for $\Delta\mu \pm E > kT$. In this case, the decay rate due to the current assumes the known value [24–26], $\Gamma_Q = (\sqrt{I_L} - \sqrt{I_R})^2/2e$. Generally, the factor $\kappa = 1 + (4t_d^2 g_\Sigma + 2\varepsilon^2 g_0)/E^2$ accounts for additional relaxation/dephasing due to particle hole excitations, induced, e.g., by thermal fluctuations of the QPC current. Finally, by introducing the phenomenological rate Γ_i we have allowed for some intrinsic charge dephasing, which occurs on the time scale of nanoseconds [28]. For an initial state in the subspace $\{|L\rangle, |R\rangle\}$, we find the stationary solution of the double dot, $\bar{\rho} = \frac{1}{2}(1 - \eta\varepsilon/\Delta\mu)|L\rangle\langle L| + \frac{1}{2}(1 + \eta\varepsilon/\Delta\mu)\times |R\rangle\langle R| - \eta(t_d/\Delta\mu)(|R\rangle\langle L| + |L\rangle\langle R|)$, where $\eta = \Gamma_Q/[\Gamma_Q(1 + g_\Sigma) + \Gamma_i]$. Positivity of $\bar{\rho}$ is satisfied since $\eta \leq \Delta\mu/E$. The time decay to $\bar{\rho}$ is described by three rates, given as the roots of $P(y) = y^3 + 2\Gamma_{\rm tot}y^2 + (E^2 + \Gamma_{\rm tot}^2)y + 4t_d^2[\Gamma_{\rm tot} + \Gamma_Q(g_\Sigma - 2g_0)\varepsilon^2/E^2]$ with $\Gamma_{\rm tot} = \kappa\Gamma_Q + \Gamma_i$. The stationary current through the QPC is given by $\bar{I} = \bar{\rho}_L I_L + \bar{\rho}_R I_R + 2et_d\lambda(\Gamma_Q/\Delta\mu){\rm Re}\bar{\rho}_{\rm RL}$ and thus becomes

$$\bar{I} = \frac{I_L + I_R}{2} + \eta\frac{\varepsilon}{2\Delta\mu}(I_R - I_L) - \eta\lambda\frac{2e\Gamma_Q t_d^2}{\Delta\mu^2}, \tag{5}$$

where $\lambda = 1 - \Delta\mu(g_- - g_+)/E$ [21]. See Ref. [23] for the current in linear response. We note that η quantifies the effect of the detuning ε on the QPC current. To reach maximal sensitivity, $\eta = 1$, we need $I_R \lesssim I_L/10$ for $I \sim$ 1nA and $\Gamma_i \sim 10^9$ s^{-1}. Note that the second term in Eq. (5) depends on ε, a property which can be used for readout, as discussed above. For example, for different Zeeman splittings and $\varepsilon_{\uparrow,\downarrow} = \pm\Delta\mu/2$, $\Gamma_i = 10^9$ s^{-1}, $I_L = 1$ nA, and $I_R = 0$, the current difference is $\bar{I}_\downarrow - \bar{I}_\uparrow = 0.4$ nA, which reduces to 0.05 nA for $I_R = 0.5$ nA. However, typical QPC currents currently reachable are $I_L = 10$ and $I_R = 9.9$ nA [17,18], i.e., the relaxation of the double dot due to the QPC is suppressed, $\eta < 10^{-3}$, and other relaxation channels become important.

Incoherent tunneling.—So far, we have discussed coherent tunneling. We can also take incoherent tunneling into account, e.g., phonon assisted tunneling, by introducing relaxation rates in Eqs. (3) and (4). For example, for detailed balance rates and neglecting coherent tunneling, we find the stationary current $\bar{I}_{\rm incoh} = \frac{1}{2}(I_L + I_R) + \frac{1}{2} \times (I_R - I_L)\tanh(\varepsilon/2kT)$ (which becomes I_R for $\varepsilon > kT$). The QPC current again depends on ε and can be used for spin readout. The current can also be measured on shorter time scales as we discuss now.

Readout with time-dependent currents is possible if there is sufficient time to distinguish I_L from I_R between two tunneling events to or from the reference dot, i.e., we consider $\Gamma_{\rm tot} > t_d$. In this incoherent regime, the tunneling from qubit to reference dot occurs with a rate $W_\uparrow$ or $W_\downarrow$, depending on the qubit state, with, say, $W_\downarrow \ll W_\uparrow$. Such rates arise from spin-dependent tunneling, $t_d^{\uparrow,\downarrow}$, or from different Zeeman splittings and tuning to tunneling resonance for, say, qubit $|\uparrow\rangle$ while qubit $|\downarrow\rangle$ is off resonant; see Figs. 1(a) and 1(b). For readout, the electron is initially on the left dot and the QPC current is I_L. Then, if the electron tunnels onto the reference dot within time t and thus changes the QPC current to I_R, such a change would be interpreted as qubit in state $|\uparrow\rangle$, otherwise as qubit $|\downarrow\rangle$. For calculating the measurement efficiency e, we note that $p_\uparrow = p_0 = 1 - e^{-tW_\uparrow}$ and $p_\downarrow = p_1 = e^{-tW_\downarrow}$ (with this type of readout, $W_\downarrow$, corresponds to a loss of the information, i.e., describes "mixing" [29]). We then maximize e by choosing a suitable t and find efficiencies $e \gtrsim 50\%$ for $W_\uparrow/W_\downarrow \gtrsim 8.75$ and $e \gtrsim 90\%$ for $W_\uparrow/W_\downarrow \gtrsim 80$.

A more involved readout is to measure the current through the QPC at different times. The current as a function of time switches between the values I_L and I_R, i.e., shows telegraph noise, as sketched in Fig. 1(d). Since the frequency of these switching events (roughly $W_\uparrow$ or $W_\downarrow$) depends on the spin, the QPC noise reveals the state of the qubit. Finally, at times of the order of the spin relaxation time T_1, the information about the qubit is lost. At each spin flip, the switching frequency changes ($W_\uparrow \leftrightarrow W_\downarrow$), which thus provides a way to measure T_1.

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